RESEARCH ARTICLE

Geometric parameters effect of the atomic force microscopy smart piezoelectric cantilever on the different rough surface topography quality by considering the capillary force

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Abstract

Nowadays, the atomic force microscopy (AFM) is widely used in the nanotechnology as a powerful nano-robot. The surface topography in Nanoscale is by far one of the most important usages of the AFM device. Hence, in this article, the vibration motion of a piezoelectric rectangular cross-section micro-cantilever (MC) which oscillates in the moist environment has been examined based on the Timoshenko beam theory. After extracting the MC governing equations according to Hamilton's principle, the finite element method has been used to discretize the motion equations. The surface topography has been simulated for various roughness forms in the tapping and non-contact modes by considering the effects of the Van der Waals, capillary and contact forces. Also, the experimental results obtained from the glass surface topography have been simulated. The results illustrate that the time delay in higher natural frequencies in the tapping mode is shorter in comparison with the non-contact mode, especially, for the lower natural frequencies. The sensitivity analysis of the natural frequencies, topography depth and time delay have been simulated. Results indicate that the most effective parameter is the MC length. In the first mode, the first section length has the highest effect on the surface topography time delay, also, in the second vibration mode; the most effective parameter on the time delay is the MC tip length based on the simulation results.

KEYWORDS

AFM piezoelectric micro-cantilever, finite element method, sensitivity analysis, Timoshenko beam, topography depth

1 | INTRODUCTION

Atomic force microscopy (AFM) is widely used in determining the surface mechanical properties, measuring the interatomic forces, and preparing 3D high resolution topographic in micro and nano scales. A cantilever with a sharp tip located at its free end, plays a pivotal role in the AFM performance. The micro-cantilever (MC) dynamic behavior is very complex and difficult. Therefore, presenting more accurate dynamic models which lead to a simpler and more accurate prediction of the system behavior is of great importance.

Today, using piezoelectric MC leads to the enhancement of speed, accuracy, and quality of the surface topography in addition to providing an economical method. As a result, the advantages of these types of cantilevers have widely attracted the researchers. With regard to research in this area, Mahmoodi et al. have studied and reviewed a partial piezoelectric MC to reduce the final cost of the AFMs construction by eliminating the need for laser (Mahmoodi, Afshari, & Jalili, 2008). In another research on this subject, they have obtained the MC first three natural frequencies by using the multi-scale method (Mahmoodi, Jalili, & Ahmadian, 2010). Fung and Huang considered the MC basis with a piezoelectric layer as an actuator and they analyzed the effect of the frequency basis on the system in the contact and non-contact modes, by using the finite element method (Fung & Huang, 2001). The MC with a partial piezoelectric layer has been investigated by Zhang, Meng, and Li (2006) and they used the Rayleigh–Ritz method for solving the
equations to reduce the degree of the differential equations and obtained results for the first 10 vibration modes of this MC. Kahrobaian, Asghari, Raahaeifard, and Ahmadian (2010) extracted the piezoelectric beam equations based on the Euler–Bernoulli beam theory and by utilizing modified couple stress (MCS). In another research, two types of AFM piezoelectric MC have been reviewed by Mahmoodi, Daqaq, and Jalili (2009) In the first model, the piezoelectric is used in the base. For the second model, the piezoelectric layer is widely used on the MC. Korayem, Korayem, and Hosseini Hashemi (2016) have simulated the piezoelectric MC by considering the Timoshenko beam theory and MCS theory in the non-contact mode and investigated the hysteresis effect on the vibration of AFM piezoelectric MCs. The nonlinear flexural vibrations of the MC with a piezoelectric layer which is placed far from the surface have considered by Nima Mahmoodi and Jalili (2007). They have used the Galerkin method in their research. Lazarus, Thomas, and Deü (2012) have investigated the MC with a piezoelectric layer in the applications of nanoelectromechanical systems. They additionally analyzed their equations by utilizing the finite element method. In another investigation conducted recently by Korayem and Nahavandi (2015), a tapping mode AFM cantilever based on the flexible beam theory with double piezoelectric layers in the presence of capillary force has been modeled and studied. Bashash et al. (2009) have investigated three types of MCS, with a piezoelectric layer, a partial piezoelectric layer, and with a single piece of piezoelectric on the free end. Reddy (2011) considered the axial force and investigated free vibration of the Euler and Timoshenko beam theory using the finite element method and the MCS theory. In another research, McCarty and Mahmoodi (2015) have studied the tapping and non-contact modes of AFM nonlinear vibrations by considering non-linear contact forces based on the Euler–Bernoulli beam theory. Li, Zhou, Zhou, and Wang (2014) have derived the governing equations for the piezoelectric Euler–Bernoulli beam by utilizing the Hamilton’s principle and the MCS theory. They have studied both static bending and free vibration problems of cantilever and simply supported micro-beams. In addition, Reinstaedtler, Rabe, Scherer, Turner, and Arnold (2003) modeled the MC as a uniform homogenous constant cross-section area, Cartesian coordinate system, and torsional and flexural vibration modes compared with the experimental results.

In recent years, numerous studies have been done to investigate the amount of AFM cantilever sensitivity to various parameters. In this regard, Rabe, Turner, and Arnold (1998) have investigated the rate of the sensitivity of the high torsional and flexural modes to the rate of sample surface stiffness. The torsional sensitivity of the V-shaped MC based on the MCS theory has been studied by Lee and Chang (2012). They considered the normal interaction between the tip and the sample surface. Farokh Payam (2013) has investigated the torsional vibration sensitivity rate of the AFM rectangular MC submerged in a fluid environment in relation to the changes of the surface stiffness. Damircheli and Korayem (2013) have analyzed the sensitivity of the higher modes for the rectangular AFM MC to the surface stiffness in the air environment in the tapping mode.

In this article, piezoelectric MC has been modeled based on the Timoshenko beam theory as a four-layer beam with a rectangular cross section including silicon cantilever, two layers of electrode made of gold (Au) and a piezoelectric layer made of ZnO enclosed by the electrodes. In order to discretize the equations, the finite element method and the Galerkin theory have been used. The frequency response and time response have been obtained far from the surface and close to the surface by considering the forces between the tip and the sample. Surface topography in the tapping and non-contact mode for rectangular and triangular roughness and a part of the profile of the glass surface has been studied. In addition, the effect of the geometric parameters on the sensitivity of the first three natural frequencies, the topography depth and the time delay in the surface topography have been investigated.

2 | THEORY

2.1 | MC governing equations based on the Timoshenko beam theory

The AFM piezoelectric MC includes four layers as in Figure 1, that, $w_1$, $w_2$, and $w_3$ are the width of the cantilever, piezoelectric layer and tip, respectively, also, $t_1$, $t_2$, $t_3$, and $t_4$ show the thickness of the cantilever, lower electrode, piezoelectric and upper electrode. A silicon cantilever, piezoelectric layer, and two layers of electrode are modeled based on the Timoshenko beam theory by considering the geometric discontinuities.

In order to extract the MC governing equations, Hamilton’s principle has been used. Given the Timoshenko beam theory, kinetic energy of each layer ($T_i$) has been calculated through Equation (1) (Harrevelt, 2012).

$$T_i = \frac{1}{2} \int_A \rho_i \left[ \left( \frac{\partial^2 w(x,t)}{\partial t^2} \right)^2 + \left( \frac{\partial^2 w(x,t)}{\partial t} \frac{\partial^2 w(x,t)}{\partial x} \right)^2 \right] dx dA$$  \hspace{1cm} (1)

where $w(x, t)$ denotes the bending displacement and $\psi(x, t)$ shows the torsion of the cross-section, $\rho_i$ shows the density of each layer and $A, (x, z)$ and $t$ show cross section area, Cartesian coordinate system components, and time, respectively. Potential energy ($U_i$) has also been calculated based on the Hooke’s law, according to Equation (2) that $K, G,$ and $E$ denote correction factor for rectangular cross section ($K = 0.83$), shear modulus and elasticity modulus (Harrevelt, 2012).

$$U_i = \frac{1}{2} \int_A \left[ E \left( \frac{\partial^2 w(x,t)}{\partial x} \right)^2 + KG \left( \frac{\partial^2 w(x,t)}{\partial x} + \psi(x,t) \right)^2 \right] dx dA$$  \hspace{1cm} (2)

For the strain energy of piezoelectric layer, the strain–stress relation has not been true based on the Hooke’s law and it must consider the electric field. Based on the available theories for piezoelectric homogeneous materials, electric displacement ($\mathbf{D}_e$, $\mathbf{D}_i$) and stress ($\mathbf{s}$, $\mathbf{s}$) for the piezoelectric layer calculated through the Equation (3); where $E_{ij}$, $\varepsilon_{ij}$, and $\phi$ show the modulus of elasticity, piezoelectric constant, dielectric constant, electric potential, strain and the electric field for the piezoelectric material, respectively (Ansari, Ashrafi, & Hosseinzadeh, 2014).
FIGURE 1  Schematic of AFM non-uniform piezoelectric MC. \( w_i, t_i, L_i \) show width, thickness and length of different layers, respectively. AFM, atomic force microscopy; MC, micro-cantilever

\[
\begin{align*}
D_i &= e_i e_{xx} + n_{t_i} \psi_s & & \sigma_x = E_{11} e_{xx} - e_1 \psi_s \\
D_2 &= e_1 e_{xx} + n_{t_2} \psi_y & & \sigma_y = E_{55} e_{xx} - e_5 \psi_y
\end{align*}
\]

\[ (3) \]

\[
\phi_x - \frac{\partial \Phi}{\partial x} \psi_x = - \frac{\partial \Phi}{\partial z} \psi_x = 0
\]

\[ (4) \]

In the absence of electric charges, electrostatic equilibrium condition is represented as \( \frac{\partial \Phi}{\partial x} + \frac{\partial \Phi}{\partial z} = 0 \) Equation (Wang & Feng, 2010). For the Timoshenko beam model, by solving this equation, with considering the boundary conditions for the piezoelectric layer including \( \psi(-\frac{a}{2}) = 0 \) and \( \psi(\frac{a}{2}) = V \) that \( V \) and \( t_3 \) denotes electric voltage and piezoelectric thickness, electric potential and stress for the piezoelectric layer have been calculated:

\[
\begin{align*}
\phi &= \left( \frac{e_1 + 2e_1}{4t_3} \right) \left( z^2 - t_3^2 \right) \left( \frac{\partial \psi(x,t)}{\partial x} \right) \\
&+ \left( \frac{e_1}{4t_3^3} \right) \left( z^2 - t_3^2 \right) \left( \frac{\partial \psi(x,t)}{\partial x} \right) + \left( z \frac{1}{t_3} + \frac{1}{2} \right) V \\
\sigma_x &= z(l + 2G) \left( \frac{\partial \psi(x,t)}{\partial x} \right) + z \left( \frac{E_1 e_3^2}{2t_3} + \frac{3}{2} \right) \left( \frac{\partial \psi(x,t)}{\partial x} \right) \\
&+ \left( \frac{e_1 e_3}{2t_3} \right) \left( \frac{\partial \psi(x,t)}{\partial x} \right) + e_1 V \\
\sigma_{zz} &= KG \left( \psi(x,t) + \frac{\partial \psi(x,t)}{\partial x} \right)
\end{align*}
\]

\[ (5) \]

As the Equation (2) for piezoelectric layer, by using the Equation (6), the potential energy of this layer has been calculated according to the Equation (7) that \( \lambda \) is the Lame constant.

\[
\begin{align*}
U_p = \int_V udV &= \frac{1}{2} \int_{x,t} \sigma \sigma \ dxdA = \int_{x,t} \frac{1}{2} \left( E + \frac{e_1 e_3}{2t_3} + 2e_1^2 \right) \left( \frac{\partial \psi(x,t)}{\partial x} \right)^2 \\
&+ \left( \frac{E_1 e_3}{2t_3} \right) \left( \frac{\partial \psi(x,t)}{\partial x} \right) + e_1 V \\
&+ KG \left( \psi(x,t) + \frac{\partial \psi(x,t)}{\partial x} \right)^2 \\
&\text{dxdA}
\end{align*}
\]

\[ (7) \]

According to the distance between the tip and the sample surface (The surface of the sample which the AFM has been taken its images to determine its dump and pump during the topography), the external force \( (F_{\text{cap}}) \) effect on the MC free end which includes the Van der Waals \( (F_{\text{vdw}}) \), capillary \( (F_{\text{cap}}) \) and contact force \( (F_{\text{DFT}}) \). Korayem, & Ghaderi, 2014) have been shown in Equation (8). Each of these forces has a repulsive or attractive essence that acts in special distance. Attractive forces can increase vibration amplitude and it is vice versa for repulsive forces. Combination of these two forces' nature, determines the amount of microcantilever amplitude and frequency at any moment.

\[
F_{\text{vdw}} = - \frac{HR}{6a_0} \left( \frac{d_{\text{off}}}{d_{\text{on}}} \right) \quad d_{\text{on}} > d_{\text{off}}
\]

\[
F_{\text{cap}} + F_{\text{DFT}} = - \frac{HR}{6a_0} \left( \frac{d_{\text{off}}}{d_{\text{on}}} \right) \left( \frac{4\pi \gamma w h}{3} \right) \quad d_{\text{off}} < d_{\text{on}}
\]

\[ (8) \]

In the above equation, \( H, R, a_0, E^*, \gamma_w \) and \( h \) are the Hamaker constant, a curvature radius of the probe tip, the moment distance of the probe tip and the sample surface, interatomic distance, effective elastic modulus, surface energy of liquid-steam and water film thickness, respectively. Under environmental conditions, a thin layer of water is always on the surface of the tip and the sample surface. As soon as the tip is reached enough to the surface, the crescent bridge forms between two films, which leads to the capillary force associated with the distance. The distance at which the bridge is formed shown with \( d_{\text{on}} \) and \( d_{\text{off}} \) represents the distance at which the bridge is broken. In order to extract the governing equations, the Hamilton’s principle has been used. By taking variation of the total kinetic energy, total potential energy and virtual work of the external forces and placing these values in the Hamilton’s principle, the MC governing equations of motion are extracted.

\[
\rho A \left( \frac{\partial^2 \psi}{\partial t^2} \right) + \left( EI + F_{\text{cap}} \left( \frac{e_1 e_3}{2t_3} + \frac{3}{2} \right) \right) \left( \frac{\partial^2 \psi}{\partial x^2} \right) \\
+ \beta \left( \frac{\partial \psi}{\partial x} \right) + F_{\text{cap}} \left( \frac{e_1 e_3}{2t_3} \right) \left( \frac{\partial^2 \psi}{\partial x^2} \right) + \gamma \delta(x - L_1) = 0
\]

\[ (9) \]

\[
\rho A \left( \frac{\partial^2 \psi}{\partial t^2} \right) + KAG \left( \frac{\partial^2 \psi}{\partial x^2} \right) + F_{\text{cap}} \left( \frac{e_1 e_3}{2t_3} \right) \left( \frac{\partial^2 \psi}{\partial x^2} \right) \\
+ C \left( \frac{\partial \psi}{\partial t} \right) - F_{\text{cap}}(L_3, t) = 0
\]

where the term \( \delta(x - L_1) \) indicates impulse function and:
\[ y = e_{1}.V.W_{3}\left( t_{1} + t_{2} + \frac{t_{3}}{2} - z_{0} \right) \]  \hspace{1cm} (11)

Constant coefficients associated with each part of the MC have been presented in Table 1. In addition, \( e_{1} \), \( e_{5} \), and \( \lambda_{33} \) are equal to \(-0.51 \text{ C/m}^2\), \(-0.45 \text{ C/m}^2\), and \(-7.88 \times 10^{-11} \text{ F/m}^3\) (Ansari et al., 2014) also \( I \) and \( z_{0} \) denote moment of inertia and neutral axis location, respectively.

2.2 Discretization of the governing equations based on the finite element method

In order to discrete the equations, the finite element method (the Galerkin theory) has been used. For this purpose, \([N]\) and \([\bar{N}]\) denote 1 x 4 matrix representing shape functions, have been considered (Lin, 1994).

\[
w = [N](P)
\]
\[
N_{1} = 1 - \frac{1}{L}(12\beta x + 3Lx^2 - 2x^3)
\]
\[
N_{2} = \frac{1}{L}(12\beta x + 3Lx^2 - 2x^3)
\]
\[
\eta = [\bar{N}](P)
\]
\[
\bar{N}_{1} = \frac{1}{L}(6x^2 - 6lx)
\]
\[
\bar{N}_{2} = \frac{1}{L}(L^2 + 12\beta)(L^2 + 12\beta - 4L^2 + 12\beta)x + 3Lx^2
\]
\[
\bar{N}_{3} = \frac{1}{L}(6Lx - 6x^2)
\]
\[
\bar{N}_{4} = \frac{1}{L}(3Lx^2 - 2Lx^2 - 12\beta x)
\]  \hspace{1cm} (12)

In which, \( \beta = \frac{El}{\text{IONG}} \) and \( P \) denotes the element nodal degrees of freedom vector including transverse displacements and rotations. In these equations, \( L, EI, K, G, A, \) and \( x \) show the length of the beam element, bending rigidity, shear coefficient, shear modulus, beam element cross-section and the coordinate along the beam element longitudinal direction, respectively. In order to discrete the equations, according to the Gelarkin method, first the residual function is calculated through using the shape function and the extracted governing equations and then by placing the residual function and the weight function in the Gelarkin equation, the mass and stiffness matrices have been obtained through calculating the obtained integrals.

The mass matrix includes two parts (\( M = M_{t} + M_{b} \)), where \( M_{t} \) represents the mass matrix for transverse inertia effects:

\[
M_{t} = \left( \frac{\rho A L}{L^2 + 12\beta} \right)
\]
\[
r_{11} \quad \cdots \quad \text{SYM}
\]
\[
r_{12} \quad \cdots \quad \text{SYM}
\]
\[
r_{13} \quad \cdots \quad \text{SYM}
\]
\[
r_{14} \quad \cdots \quad \text{SYM}
\]  \hspace{1cm} (14)

where

\[
r_{12} = r_{21} = \frac{13}{35}L^4 + \frac{42}{5}L^2 + 48\beta^2
\]
\[
r_{13} = -r_{23} = \left( \frac{11}{210}L^4 + \frac{11}{10}L^2 + 6\beta^2 \right)L
\]
\[
r_{14} = -r_{24} = \left( \frac{1}{140}L^4 + \frac{9}{5}L^2 + \frac{6\beta^2}{5} \right)L^2
\]  \hspace{1cm} (15)

\[ M_{r} \] describes the rotary inertia effect:

\[
M_{r} = \left( \frac{\rho L}{L^2 + 12\beta} \right)
\]
\[
r_{11} \quad \cdots \quad \text{SYM}
\]
\[
r_{12} \quad \cdots \quad \text{SYM}
\]
\[
r_{13} \quad \cdots \quad \text{SYM}
\]
\[
r_{14} \quad \cdots \quad \text{SYM}
\]  \hspace{1cm} (16)

where

\[
r_{11} = r_{33} = \frac{6L^4}{5}
\]
\[
r_{12} = r_{41} = -r_{22} = \left( \frac{1}{10}L^2 - 6\beta^2 \right)L^3
\]
\[
r_{31} = r_{42} = -r_{21} = \left( \frac{1}{5}L^4 - 2L^2 + 4\beta^2 \right)L^2
\]  \hspace{1cm} (17)

Stiffness matrix also includes bending stiffness with in addition to shear effect which has been shown in the Equation (19).

\[ k = k_{b} + k_{s} \]

\[
k_{b} = \left( EI + IP \left( \frac{e_{1} e_{5} + e_{3}^2}{2\lambda_{33}} \right) \right) \left( \frac{12}{L^2 (L^2 + 12\beta)^2} \right)
\]
\[
t_{11} \quad \cdots \quad \text{SYM}
\]
\[
t_{21} \quad t_{22} \quad \cdots \quad \text{SYM}
\]
\[
t_{31} \quad t_{32} \quad t_{33} \quad \cdots \quad \text{SYM}
\]
\[
t_{41} \quad t_{42} \quad t_{43} \quad t_{44} \]  \hspace{1cm} (18)

\[
k_{s} = \left( IP \left( \frac{e_{1} e_{5}}{2\lambda_{33}} \right) \right) \left( \frac{12}{L^2 (L^2 + 12\beta)^2} \right)
\]
\[
t_{11} \quad t_{21} \quad t_{22} \quad \cdots \quad \text{SYM}
\]
\[
t_{31} \quad t_{32} \quad t_{33} \quad \cdots \quad \text{SYM}
\]
\[
t_{41} \quad t_{42} \quad t_{43} \quad t_{44} \]
where

\[
\begin{align*}
    t'_{11} &= t'_{33} = -t'_{31} = 1 & t_{11} &= t_{33} = -t_{31} = L^3 - 12\mu L \\
    t'_{21} &= t'_{41} = \frac{L}{2} & t_{21} &= t_{41} = -t_{43} = \frac{L^4}{2} \\
    t'_{22} &= \frac{L^2}{6} + \beta & t_{22} &= t_{44} = \frac{L^5}{3} + 12\mu^2 L + 2\mu L^2 \\
    t'_{42} &= \frac{L^2}{6} - \beta & t_{42} &= \frac{L^5}{2} - 2\mu L^2 - 12\mu^2 L
\end{align*}
\]  

(19)

By considering the mass and stiffness matrix for each element, these matrices were assembled and general MC mass and stiffness matrices have been obtained and applied in Equation (21) that in this equation \( F_{\text{piezo}} = \int_0^1 N^T(x) : \bar{\alpha}(x - L_1)dx. \)

\[
[M] \{\ddot{P}\} + [C] \{\dot{P}\} + [K] \{P\} = F_{\text{piezo}} + F_{\text{eq}}
\]

(20)

Natural frequencies (\(\omega_n\)) and mode shapes (\(\phi_n\)) are respectively obtained from \([K - \omega_n^2M]\) and \([K - \omega_n^2C]\) \(\{\phi\} = 0\). Damping matrix \(C\) has been modeled in modal space by using eigenvectors for natural frequencies and the quality factor \(Q\) (Korayem et al., 2014). In order to calculate the frequency response, by using the Laplace transform and transferring the motion equation from the time domain to the frequency domain, the MC frequency response has been calculated according to Equation (22).

\[
\{TF(\omega)\} = \{-\omega^2[M] + j\omega[C] + [K]\}^{-1} \{F(\omega)\}
\]

(21)

The MC time response has been calculated based on the Newmark algorithm. This method divides the time interval to the smaller steps (\(\Delta t\)) and in each step, it is assumed that the acceleration of each degree of freedom is constant and the system state (acceleration, velocity, and displacement vector) is known at time \(t\) and the displacement, velocity and acceleration vector is unknown at time \((t + \Delta t)\). This process is repeated until the end of the simulation time. The \(\hat{K}\) matrix is constant and it is enough to calculate it.

### 3 RESULTS AND DISCUSSION

#### 3.1 The MC vibration behavior in the vicinity of sample surface

In order to simulate piezoelectric MC, the MATLAB software has been used and the constants for the MC with the piezoelectric layer are shown in Table 2 (Nima Mahmooodi & Jalili, 2007).

When the MC is far from the surface, no force is applied to the probe. The results obtained for the frequency response in the free vibration mode have been compared with the experimental results (Nima Mahmooodi & Jalili, 2007) and the results obtained based on the Euler–Bernoulli beam theory (Korayem et al., 2014). As it is shown in Table 3, the higher natural frequency is the more error percentage will be and by comparing the obtained results, the Timoshenko beam theory compiles more with the experimental results and the error percentage of the Timoshenko beam theory was lower in comparison with the Euler–Bernoulli beam theory. For the first frequency, the error value has been reduced from 0.14% to 0.017%. It's noticeable that this work trying to find the lowest error for both Euler and Timoshenko beam theories. For this aim, this article found the best value in comparison with the Euler theory by considering the inertial terms and the rotation of the beam area cross section which is the main reason why the Timoshenko beam theory has less error in comparison with the Euler theory.

Figure 2 shows the comparison of the piezoelectric MC time response with the experimental results and the Euler–Bernoulli beam theory, by applying the sinusoidal voltage with the amplitude of 1.5 V, in the first mode. The results obtained from the simulation strongly comply with the experimental results which show the high accuracy of the Newmark algorithm in solving the dynamic equation, also the fact that Timoshenko beam theory is more accurate in proportion to the Euler–Bernoulli beam theory. The MC vibration amplitude, which has been modeled based on the Timoshenko beam theory, has been calculated 3.78 \(\mu m\) which has an error of \(-0.8\%\) in comparison with

### Table 1 Coefficients of Equations (9) and 10 for each section of the MC

<table>
<thead>
<tr>
<th>Section</th>
<th>(EI)</th>
<th>(\beta)</th>
<th>(\rho l)</th>
<th>(\rho A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 &lt; x &lt; L_1)</td>
<td>(\sum_{i=1}^{d} E_i I_i)</td>
<td>(\sum_{i=1}^{d} \beta_i)</td>
<td>(\sum_{i=1}^{d} \rho_i l_i)</td>
<td>(\sum_{i=1}^{d} \rho_i A_i)</td>
</tr>
<tr>
<td>(L_1 &lt; x &lt; L_2)</td>
<td>(E_{L_1})</td>
<td>(\rho_{L_1})</td>
<td>(\rho_{L_1} L_{L_1})</td>
<td>(\rho_{L_1} A_{L_1})</td>
</tr>
<tr>
<td>(L_2 &lt; x &lt; L_3)</td>
<td>(E_{L_2})</td>
<td>(\rho_{L_2})</td>
<td>(\rho_{L_2} A_{L_2})</td>
<td>(\rho_{L_2} A_{L_2})</td>
</tr>
</tbody>
</table>

Abbreviation: MC, micro-cantilever.

### Table 2 Geometric constants for simulation the rectangular MC (Korayem et al., 2014)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Material</th>
<th>(L)</th>
<th>(W)</th>
<th>(t)</th>
<th>(\mu)</th>
<th>(E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit</td>
<td>(\mu m)</td>
<td>(\mu m)</td>
<td>(\mu m)</td>
<td>(kg/m(^2))</td>
<td>(Gpa)</td>
<td></td>
</tr>
<tr>
<td>Cantilever</td>
<td>Si</td>
<td>375 ± 5 (\mu m)</td>
<td>250 ± 5 (\mu m)</td>
<td>4 ± 0.5 (\mu m)</td>
<td>2,330</td>
<td>105</td>
</tr>
<tr>
<td>Lower electrode</td>
<td>Au</td>
<td>330 ± 5 (\mu m)</td>
<td>130 ± 5 (\mu m)</td>
<td>0.25</td>
<td>19,300</td>
<td>78</td>
</tr>
<tr>
<td>Piezoelectric</td>
<td>ZnO</td>
<td>330 ± 5 (\mu m)</td>
<td>130 ± 5 (\mu m)</td>
<td>4 ± 0.5 (\mu m)</td>
<td>6,390</td>
<td>104</td>
</tr>
<tr>
<td>Upper electrode</td>
<td>Au</td>
<td>330 ± 5 (\mu m)</td>
<td>130 ± 5 (\mu m)</td>
<td>0.25</td>
<td>19,300</td>
<td>78</td>
</tr>
<tr>
<td>Tip</td>
<td>Si</td>
<td>125 ± 5 (\mu m)</td>
<td>55 ± 5 (\mu m)</td>
<td>4 ± 0.5 (\mu m)</td>
<td>2,330</td>
<td>105</td>
</tr>
</tbody>
</table>

Abbreviation: MC, micro-cantilever.
the amplitude of 3.75 μm, obtained from the experiment; whereas the error value reported for the results obtained from the Euler–Bernoulli beam theory had been 1.6%. Therefore, as it is observed, using the Timoshenko beam theory has increased the accuracy of the modeling to a large extent.

As the piezoelectric MC gets closer to the surface, the effects of the forces between the tip and the sample on the MC vibration behavior, increases. Absolutely viscosity of the working environment is one of the most prominent factors which play a great role in the amount of MC amplitude. This parameter due to a negligible amount in the air environment which resulting no tangible effect in the aforementioned situation (for instance ambient temperature) in comparison to the other great forces like Van der Waals which is not considered in this article. While, in the liquid environment it declines cantilever amplitude remarkably by increasing ambient damping (Korayem, Taghizade, & Korayem, 2018). Since viscosity is affected by temperature, all the results and simulations should be in the certain temperature. Also, the temperature changes the haymaker ratio which is affected the capillary and Van der Waals force, but this amount of change is very small. Table 4 shows the first natural frequency and the amplitude of the vibration motion increases as well. The changes of the sample surface and as the equilibrium distance increases, the second frequency mode.

For the sample surface topography, by piezoelectric MC, the MC time response changes when it passes the surface roughness that the scan speed is considered about 1 μm/s. Two types of the surface roughnesses are simulated. The rectangular roughness considered as a rectangular hole which is shown in the figures. In addition, the triangular roughness is considered as the triangle with the specific height which is also shown in the figures. The force constants used here include water vapor–liquid energy (γw), radius of the probe (R), interatomic distance (d0), water layer thickness (h), capillary bridge start (dh), breaking capillary bridge (dμ), the Hamaker coefficient (H), Needle Young’s modulus (Ei), sample Young’s modulus (Es), the MC Poisson coefficient (ν), the sample Poisson coefficient (νs), and quality factor (Q) which are equal to 75 μJ/m², 20 nm, 0.34 nm, 0.2 nm, 0.4 nm, 2.32 nm, 6x10⁻²⁰ J, 105 Gpa, 78 Gpa, 0.287, 0.287 and 167, respectively (Korayem et al., 2014). At first, the rectangular and the triangular shaped roughness in the non-contact mode has been considered. Simulations for the first three modes were studied and the results obtained from the first two modes have been compared with the results obtained from the Euler–Bernoulli beam theory. Also, in the tapping mode, the rectangular and the triangular roughness in the first and the second natural frequency have been simulated and the results have been compared with the non-contact mode.

3.2 Sample surface topography in the amplitude mode

For the sample surface topography, by piezoelectric MC, the MC time response changes when it passes the surface roughness that the scan speed is considered about 1 μm/s. Two types of the surface roughnesses are simulated. The rectangular roughness considered as a rectangular hole which is shown in the figures. In addition, the triangular roughness is considered as the triangle with the specific height which is also shown in the figures. The force constants used here include water vapor–liquid energy (γw), radius of the probe (R), interatomic distance (d0), water layer thickness (h), capillary bridge start (dh), breaking capillary bridge (dμ), the Hamaker coefficient (H), Needle Young’s modulus (Ei), sample Young’s modulus (Es), the MC Poisson coefficient (ν), the sample Poisson coefficient (νs), and quality factor (Q) which are equal to 75 μJ/m², 20 nm, 0.34 nm, 0.2 nm, 0.4 nm, 2.32 nm, 6x10⁻²⁰ J, 105 Gpa, 78 Gpa, 0.287, 0.287 and 167, respectively (Korayem et al., 2014). At first, the rectangular and the triangular shaped roughness in the non-contact mode has been considered. Simulations for the first three modes were studied and the results obtained from the first two modes have been compared with the results obtained from the Euler–Bernoulli beam theory. Also, in the tapping mode, the rectangular and the triangular roughness in the first and the second natural frequency have been simulated and the results have been compared with the non-contact mode.

Figure 3 shows the piezoelectric MC topography while passing the rectangular and the triangular roughness for the Euler–Bernoulli (Korayem et al., 2014) and Timoshenko beam in the first and the second frequency mode.

As it has been shown, when the probe passes the roughness of the sample surface and as the equilibrium distance increases, the amplitude of the vibration motion increases as well. The changes of the MC amplitude during scanning the rectangular and the triangular roughness are slow due to the high inertia in the first mode. However,

TABLE 3 Comparing the frequency response, far from the sample surface, with the experimental results and the Euler–Bernoulli beam theory

<table>
<thead>
<tr>
<th>Natural frequency</th>
<th>Euler–Bernoulli (kHz)</th>
<th>Timoshenko (kHz)</th>
<th>Experiment (kHz)</th>
<th>Error percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>52.377</td>
<td>52.291</td>
<td>52.31</td>
<td>0.14%</td>
</tr>
<tr>
<td>Second</td>
<td>203.414</td>
<td>203.075</td>
<td>203.0</td>
<td>0.20%</td>
</tr>
<tr>
<td>Third</td>
<td>392.375</td>
<td>391.632</td>
<td>382.5</td>
<td>2.5%</td>
</tr>
</tbody>
</table>

Error percentage

<table>
<thead>
<tr>
<th></th>
<th>Euler–Bernoulli</th>
<th>Timoshenko</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler–Bernoulli</td>
<td>0.14%</td>
<td>0.017%</td>
</tr>
<tr>
<td>Timoshenko</td>
<td>0.20%</td>
<td>0.03%</td>
</tr>
<tr>
<td>2.5%</td>
<td>2.3%</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 4 Comparing the MC frequency and amplitude at different equilibrium distance with the results based on the Euler–Bernoulli theory

<table>
<thead>
<tr>
<th>Base distance from surface</th>
<th>Timoshenko</th>
<th>Euler–Bernoulli</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude (nm)</td>
<td>Frequency (kHz)</td>
<td>Amplitude (nm)</td>
</tr>
<tr>
<td>Far from surface</td>
<td>3.001</td>
<td>55.15</td>
</tr>
<tr>
<td>5 nm</td>
<td>2.892</td>
<td>54.99</td>
</tr>
<tr>
<td>4 nm</td>
<td>2.761</td>
<td>54.83</td>
</tr>
<tr>
<td>3 nm</td>
<td>2.468</td>
<td>54.40</td>
</tr>
<tr>
<td>2 nm</td>
<td>2.158</td>
<td>52.47</td>
</tr>
</tbody>
</table>

Abbreviation: MC, micro-cantilever.
in the second mode, in which the motion of the MC during the scanning the sample surface with a higher frequency, the velocity of the tip encountering the sample surface is higher and therefore the response to the changes of the surface is also quicker. Thus, the shape of the roughness has been shown more accurately. By comparing the results in Table 5, it could be observed that since the changes of the surface are scanned gradually in the triangular roughness and there are no sudden changes as there are in the rectangular roughness, the time delay in recognizing the maximum depth of the roughness is shorter. The second mode shows the changes of the surface roughness more accurately in comparison with the first mode and the time delay for the model based on the Timoshenko theory is shorter than the model based on the Euler–Bernoulli beam theory in both modes. In total, the shortest time delay is associated with the second mode and modeling based on the Timoshenko beam theory, which is indicative of the better accuracy of the Timoshenko beam theory in the topography of the surfaces in comparison with the Euler–Bernoulli beam theory.

Figure 4 shows the time response changes when the piezoelectric MC is scanning the rectangular roughness in the third mode in comparison with the first two modes. As it is shown, in higher modes, the depth of the roughness has been shown lower but since the natural frequency is higher in the third mode in proportion to the first two modes, therefore the time delay has been reduced in comparison with those two modes and the cantilever has reached the maximum of its amplitude changes in shorter time interval. For the rectangular roughness, the rate of time delay in the first, second and the third mode are 0.81, 0.78, and 0.75 ms, respectively; whereas, for the triangular roughness, due to the gradual changes of the surface in comparison with the rectangular roughness, the delay time is shorter in each three vibration modes which are 0.77, 0.59, and 0.55 ms, respectively.

The two-dimensional (2D) image obtained from the part of the glass surface topography has been shown in Figure 5b and the roughness parameters of the sample surface for this sample have been extracted. Topography has been taken by Dual Scope DS 95-50-E AFM. The MC has aluminum coatings and the tip of the probe is made up out of silicon. The images have been taken at the resonance frequency of 190 kHz and the scan size of 30 × 15 μm. Surface roughness parameters including the maximum height roughness root mean square roughness and roughness height standard.

Figure 5c illustrates the profile of a part of the glass surface along the green line. Figure 5a indicates the results of the topography simulation for the considered profile. As it has been shown, the more the slope of the changes of the surface roughness is, the longer the delay will be in recognizing the changes of the surface roughness.

For the simulation of the surface topography in the tapping mode, first the vibration amplitude is adjusted in ~30 nm while the MC tip does not have contact with the surface; then, as the MC is taken closer to the sample surface it is adjusted in a way that the probe tip enters the sample for 5 nm. For this purpose, the distance of the MC basis is adjusted to be 25 nm. Since the repulsive force $F_{DMT}$ prevents the probe tip from immerging in the sample, the depth of the penetration is obtained to be <5 nm. In this article, topography for the rectangular and the triangular roughness has been simulated for the first and the second modes and then the obtained results have been compared with the results obtained from the non-contact mode. The considerable point, in surface topography, is the presence of a time difference for recognizing the starting point of the surface roughness. This time delay causes the recognition and the topography of the nanoparticle status that exists in the sample surface to have low accuracy. Figure 6 shows the surface topography in the tapping mode for the rectangular and the triangular roughness. As it has been shown, in both of the first and the second modes, the time delay is shorter in the tapping mode in comparison with the non-contact mode (for rectangular roughness in tapping mode the time delay for the first mode and second mode is 1.57 and 0.57 ms and for triangular roughness is 0.51 and 0.39 ms, respectively) and in the tapping mode, just like the non-contact one.

**TABLE 5** Comparing the time delay in the detection of surface roughness

<table>
<thead>
<tr>
<th>Theory</th>
<th>Time delay – first mode (ms)</th>
<th>Error percentage – first mode</th>
<th>Time delay – second mode (ms)</th>
<th>Error percentage – second mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rectangular</td>
<td>Triangle</td>
<td>Rectangular</td>
<td>Triangle</td>
</tr>
<tr>
<td>Timoshenko</td>
<td>1.76</td>
<td>0.77</td>
<td>29.33%</td>
<td>11.84%</td>
</tr>
<tr>
<td>Euler–Bernoulli</td>
<td>1.87</td>
<td>0.81</td>
<td>31.16%</td>
<td>12.46%</td>
</tr>
</tbody>
</table>
in higher natural frequency, the topography is more accurate and roughness has more properly shaped.

Finally, as it is shown, using the Timoshenko beam theory causes the more accuracy in the simulation results, including the frequency response and the surface topography results. The more accurate simulation and modeling show their importance when this article uses the simulation results to predict the piezoelectric MC vibration behavior to improve the quality of the topography images by providing examine the environmental forces effect on the transient response. Also in this regard, using the sensitivity analysis methods has greater importance which is investigated in Section 5. All in all, this method helps users by more precise prediction of the cantilever vibration motion and ability of destructive forces elimination like capillary force, increase the quality of topography images (Korayem, Taghizadeh, Abdi, & Korayem, 2018). Preparing topography images of soft samples surface which their combinations are changed by time but do not have any deformation (disturbs the adjusted equilibrium distance) can be possible by this method because of momentary data gathering. However, most of the soft samples like biology samples should be preserved in the fluid environment during the test to be remained alive without structural alterations, so the simulation method will become more complicated and may be considered the effect of the new ambient. It is a consideration that MC should be set in the special voltage due to its vibration near the resonance frequency far from the surface. Based on the presented formulation it is possible too changed the voltage for a different distance, different cantilever and sample, this article presents general equation but for the simulation, it is possible to change every input parameter including the voltage based on the experimental result and AFM set points.

3.3 The sensitivity analysis of the MC frequency response and topography parameters

The sensitivity analysis is studied to simplify the model and present the relation between the input and output data. It is also predicted, what kind of response the model gives to the input variables and their changes. Given that the equations presented for the simulation of a model are mostly nonlinear and also due to being coupled, calculating the effect of the parameter interaction in the model outputs is prominent along with calculating the main effect of each parameter. Therefore, the method which is independent of the type of the equations (linear and nonlinear) can calculate the interaction between the parameters will be an ideal method for predicting the behavior of parameters in the equations. The methods based on variance have such features and that is why they are the most applicable methods in various research fields. Sobol’s method is one of these methods.

FIGURE 4 Surface topography in the first three vibration mode of the piezoelectric MC, (a) triangular roughness and (b) rectangular roughness. MC, micro-cantilever

FIGURE 5 Topography of the glass surface, (a) simulation profiles of glass surface, (b) two-dimensional topography of the glass surface, and (c) glass surface profile along the green line
However, given that in some cases the number of the modeling parameters are too numerous, Sobol’s method is not proper for reviewing the equations because the number of samples it considers for investigating the sensitivity of the parameters increases and the process of the sensitivity analysis with this method takes a lot of time. Thus, in this article, Efast sensitivity analysis method has been used along with Sobol method. This method is also based on variance like Sobol method. Its advantage compared to Sobol method is that the number of data it considers for sensitivity analysis is lower in comparison with Sobol method. In this article, the effect of nine MC geometric parameters, which includes the length of the MC first part ($L_1$), the length of the MC middle part ($L_2 - L_1$), the tip length ($L_3 - L_2$), the thickness of the MC, piezoelectric layer and electrodes and the width of the MC, piezoelectric layer and the tip on the first, second and third natural frequency have been examined by using Sobol method and Efast. Also, the effect of geometric parameters on the topography depth and the time delay in recognizing the maximum roughness depth has been investigated.

Figure 7 shows how the frequency of the first three resonance modes is affected by the thickness of the silicon, piezoelectric layer and electrodes. Since as each of the layers thickness increases it adds to the MC stiffness, therefore increasing the thickness will lead to the enhancement of the natural frequency. Given the obtained results, the MC thickness and the piezoelectric layer, due to being thicker, causes the natural frequency to go through more changes. Since the electrodes are thinner, they have less impact on the MC stiffness. Therefore, increasing their thickness does not cause many changes in the natural frequency. In addition, the results indicate the effect of the piezoelectric layer thickness on the first frequency is more than the others; whereas, for higher modes, the graph slope is more for the MC thickness.
In Figure 8, the results obtained from sensitivity analysis using Sobol and Efast method for the first mode have been shown that are indicative the frequency sensitivity percentage of each geometrical parameters. As it is observed, the results obtained from Sobol’s method and Efast method are almost similar and their difference is lower than 5%.

The results obtained from the sensitivity analysis for the second and the third modes have also been shown in Table 6. The results indicate that the most sensitive parameter is the length. The MC first part length has the highest effect on the first natural frequency. The MC middle part length has the lowest effect on the frequency in comparison with the other two parts, because of its shortness. Generally, the results indicate that the wideness of the layers impact on the natural frequency and also the effect of the piezoelectric layer thickness are higher in the first mode in comparison with the other two.

<table>
<thead>
<tr>
<th>Method</th>
<th>$L_1$</th>
<th>$L_2-\text{L}_1$</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$t_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2nd</td>
<td>3rd</td>
<td>2nd</td>
<td>3rd</td>
<td>2nd</td>
<td>3rd</td>
<td>2nd</td>
<td>3rd</td>
</tr>
<tr>
<td>Efast</td>
<td>21</td>
<td>24</td>
<td>5</td>
<td>8</td>
<td>53</td>
<td>43</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Sobol</td>
<td>22</td>
<td>23</td>
<td>5</td>
<td>5</td>
<td>56</td>
<td>46</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

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The considerable point in the sample surface topography is the accuracy in recognizing the maximum roughness depth. Figure 9 shows how the recognized slope of triangular roughness is affected by the length of each section including the MC first section length, the MC second section length and the tip length. As the length of the MC increases, its total mass and stiffness also increase and generally, the MC inertia reduces. Inertia is the tendency of the objects to maintain their previous state. On the other hand, in the self-actuated MC, as the piezoelectric layer length increases, the intensity of the actuating force of the MC also increases. Therefore, as the length increases, the roughness depth increases. In addition, to the longer length of the first part, the effects of $L_1$ is greater and the graph is more sloped as well.

In Figure 10, the results obtained from sensitivity analysis have been presented by using Efasm method for the first and the second modes. As it is observed, both modes show that the MC middle part length has less effect on the slope of roughness in comparison with the other two parts since it is shorter. Also, the MC thickness has a considerable effect on the amplitude and the wideness parameter has the least impact on the roughness slope. In addition, the results show that the most sensitive parameter in the first mode is $L_1$ and in the second mode, it is the length of the tip.

Another considerable point in the sample surface topography is the presence of a time difference for recognizing the starting point of the surface roughness changes. The time delay increases when the MC passes a more sloped roughness and more severe abrupt changes. As the time delay is longer, the less accurate the recognition and topography of the status of the nanoparticle in the sample surface will be. Thus, examining the effect of the geometrical parameters on the time delay is crucially important.

Figure 11 described how the time delay, for triangular roughness, in the first and second resonant modes is affected by geometric parameters. As it is observed, the most sensitive parameter is the length. In the first mode, the length of the first part has the greatest effect and in the second mode, the most effective parameter is the tip length. Generally, the results indicate that the wideness of the layers has less effect on the natural frequency and therefore on the time delay.

4 | CONCLUSION

In this article, the vibration behavior of the AFM piezoelectric MC with a rectangular cross-section has been the focus of the research based on the Timoshenko beam theory. The finite element method has been used for discretizing the equations extracted based on Hamilton's principle. The MC frequency and time response in distances far from the surface and close to the surface have been simulated. For the rectangular and wedge-shaped roughness in the non-contact and tapping modes and also for a part of the profile and surface roughness of the glass topography has been investigated. In addition, sensitivity analysis has been utilized for the first three natural frequencies, the topography depth, and also for the calculation of the rate of the time delay in the surfaces topography with wedge-shaped roughness for the first two modes.

By comparing the obtained frequency and time response with the experimental results and the finite element method, it was shown that modeling based on the Timoshenko beam theory, by considering the effects caused by shearing, has led to the improvement of the results. The results obtained from topography show that higher modes are more accurate in the surface topography. Additionally, it was observed that the more gradual the changes of the surface are, the shorter the delay in recognizing the surface roughness will be. In addition, by comparing the topography results in the tapping mode with the non-contact mode, it was shown that the time delay is reduced and the roughness shape is shown more accurately. The results
obtained from sensitivity analysis also illustrate that the most effective parameters on the first three frequencies are the topography depth, the time delay and length. Finally, the least effective one is the width of the layers. In the first frequency, the length of the first part of the MC is the most effective parameter and in higher frequencies, the maximum effect was the tip length.

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APPENDIX A

\[
\epsilon_{xx} = 2 \frac{\partial \psi(x,t)}{\partial x}, \quad \epsilon_{zz} = \frac{1}{2} \left( \frac{\partial \psi(x,t) + \partial \phi(x,t)}{\partial x} \right)
\]

\[
\begin{cases}
D_1 = e_{54xz} + \eta_{1100}

D_2 = e_{15xz} + \eta_{2300}
\end{cases}
\]
\[ \varphi_x = \frac{\partial \varphi}{\partial x} \varphi_z = -\frac{\partial \varphi}{\partial z} \varphi_y = 0 \]
electric potential along the micro-beam (x-axis) except in the vicinity of two ends is almost constant which results in \( \varphi_x \leq \varphi_y \) (Wang & Song, 2006), by considering this point:

\[
\begin{align*}
D_x &= e_1 \left( \frac{1}{2} \left( \psi(x, t) + \frac{\partial \psi(x, t)}{\partial x} \right) \right) - \lambda_{11} \left( \frac{\partial \varphi}{\partial x} \right) \\
D_z &= e_1 \left( \frac{2}{3} \left( \frac{\partial \psi(x, t)}{\partial x} \right) \right) - \lambda_{33} \left( \frac{\partial \varphi}{\partial z} \right)
\end{align*}
\]

In the absence of electric charges, electrostatic equilibrium condition is represented as \( \frac{\partial \varphi}{\partial x} + \frac{\partial \varphi}{\partial z} = 0 \). So

\[ e_1 \left( \frac{\partial^2 \psi(x, t)}{\partial x^2} + \frac{\partial^2 \psi(x, t)}{\partial z^2} \right) + 2e_1 \left( \frac{\partial^2 \psi(x, t)}{\partial x \partial z} \right) - \lambda_{33} \left( \frac{\partial \varphi}{\partial z} \right) = 0 \]

\[ \left( \frac{\partial^2 \varphi}{\partial x^2} \right) = e_3 + \frac{e_2}{4\lambda_{33}} \left( \frac{\partial \psi(x, t)}{\partial x} \right) + e_1 \left( \frac{\partial^2 \psi(x, t)}{\partial x^2} \right) \]

\[ \varphi = e_3 + \frac{e_2}{4\lambda_{33}} \left( \frac{\partial \psi(x, t)}{\partial x} \right) + e_1 \left( \frac{\partial^2 \psi(x, t)}{\partial x^2} \right) \]

\[ \varphi = e_3 + \frac{e_2}{4\lambda_{33}} \left( \frac{\partial \psi(x, t)}{\partial x} \right) + e_1 \left( \frac{\partial^2 \psi(x, t)}{\partial x^2} \right) \]

By considering boundary condition: \( \varphi \left( -\frac{t_2}{2} \right) = 0 \) and \( \varphi \left( \frac{t_2}{2} \right) = V \)

\[ \varphi = e_3 + \frac{e_2}{4\lambda_{33}} \left( \frac{\partial \psi(x, t)}{\partial x} \right) + e_1 \left( \frac{\partial^2 \psi(x, t)}{\partial x^2} \right) \]

Stresses for a piezoelectric micro-beam are obtained as follows (Wang & Song, 2006)

\[ \sigma_x = E_{11} e_x - e_1 \varphi_z \]
\[ \sigma_z = E_{55} e_z - e_3 \varphi_x \]
\[ \sigma_x = z(\lambda + 2G) \left( \frac{\partial \psi(x, t)}{\partial x} \right) + z \left( \frac{e_1 e_5}{4\lambda_{33}} + \frac{e_1}{\lambda_{33}} \right) \left( \frac{\partial \psi(x, t)}{\partial x} \right) \]
\[ + z \left( \frac{e_2}{4\lambda_{33}} \right) \left( \frac{\partial^2 \psi(x, t)}{\partial x^2} \right) + e_1 \frac{V}{t_3} \]
\[ \sigma_z = KG \left( \psi(x, t) + \frac{\partial \psi(x, t)}{\partial x} \right) \]

Governing equation of motion:

\[ \int_{t_1}^{t_2} (\partial T - \partial U + \partial W) dt = 0 \]

\[ T_1 = \frac{1}{2} \int_{t_1}^{t_2} \rho \left[ \frac{\partial (\psi(x, t))}{\partial t} \right]^2 + \left( \frac{z \psi(x, t)}{\partial t} \right)^2 \] dxdA

\[ T_{total} = \frac{1}{2} \int_{t_1}^{t_2} \rho \left( \frac{\partial \psi(x, t)}{\partial t} \right)^2 + \rho A \left( \frac{\partial \psi(x, t)}{\partial t} \right)^2 \] dxdA

\[ \partial T = \int_{t_1}^{t_2} \left[ \rho A \left( \frac{\partial \psi(x, t)}{\partial t} \right) \partial \left( \frac{\partial \psi(x, t)}{\partial t} \right) + \rho \left( \frac{\partial \psi(x, t)}{\partial t} \right) \partial \left( \frac{\partial \psi(x, t)}{\partial t} \right) \right] dxdA \]

For the micro-cantilever and two electrode layers:

\[ U = \int_{t_1}^{t_2} \left[ E \left( \frac{\partial \psi(x, t)}{\partial x} \right)^2 + KG \left( \frac{\partial \psi(x, t)}{\partial x} + \psi(x, t) \right)^2 \right] dxdA \]

\[ \delta U = \int_{t_1}^{t_2} \left( \frac{e_2}{2\lambda_{33}} \left( \frac{\partial \psi(x, t)}{\partial x} \right) + e_1 \frac{V}{t_3} \left( \frac{\partial \psi(x, t)}{\partial x} \right) \right) \partial \left( \frac{\partial \psi(x, t)}{\partial x} \right) \] dxdA

For piezoelectric layer

\[ U_p = \int_{t_1}^{t_2} \left( \frac{e_1 e_5}{2\lambda_{33}} \left( \frac{\partial \psi(x, t)}{\partial x} \right) \right) dxdA \]

\[ U_p = \frac{1}{2} \left( I_p \left( E_p + e_1 e_5 + 2e_1^2 \right) \left( \frac{\partial \psi(x, t)}{\partial x} \right) \right) \]

\[ + I_p \left( e_1 e_5 \frac{\partial^2 \psi(x, t)}{\partial x^2} \right) \left( \frac{\partial \psi(x, t)}{\partial x} \right) \]

\[ + KG \left( \psi(x, t) + \frac{\partial \psi(x, t)}{\partial x} \right)^2 \]

\[ + e_1 V \left( W_3 \left( t_1 + t_2 + \frac{t_3}{2} \right) - Z_n \right) \left( H(x-L_1) \right) \]

\[ \partial U_p = \int_{t_1}^{t_2} \left( I_p \left( E_p + e_1 e_5 + 2e_1^2 \right) \left( \frac{\partial \psi(x, t)}{\partial x} \right) + I_p \left( e_1 e_5 \frac{\partial^2 \psi(x, t)}{\partial x^2} \right) \left( \frac{\partial \psi(x, t)}{\partial x} \right) \right) \]

\[ + KG \left( \psi(x, t) + \frac{\partial \psi(x, t)}{\partial x} \right) \delta \left( \psi(x, t) + \frac{\partial \psi(x, t)}{\partial x} \right) \]

\[ + e_1 V \left( W_3 \left( t_1 + t_2 + \frac{t_3}{2} \right) - Z_n \right) \left( H(x-L_1) \right) \] dxdA

\[ \partial W = \int_{t_1}^{t_2} \left[ \partial \psi(x, t) \left( \frac{\partial \psi(x, t)}{\partial x} \right) \right] dxdA \]

According to Hamilton principle and by considering C for system damping, governing equation is as follows:

\[ \rho \left( \frac{\partial^2 \psi}{\partial t^2} \right) + \left( EI + I_p \left( e_1 e_5 \frac{\partial^2 \psi}{\partial x^2} \right) \right) \left( \frac{\partial \psi}{\partial x} \right) \]

\[ + \rho \left( \psi + \frac{\partial \psi}{\partial x} \right) + I_p \left( e_1 e_5 \frac{\partial^2 \psi}{\partial x^2} \right) + \gamma \left( \frac{\partial \psi}{\partial x} \right) \]

\[ + \gamma \left( \frac{\partial \psi}{\partial x} \right) \]

\[ + \gamma \left( \frac{\partial \psi}{\partial x} \right) \]