Sensitivity analysis of non-uniform piezoelectric micro-cantilever amplitude in different humidity percentages

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Abstract:
Atomic force microscopy (AFM) is a pioneer and powerful tool in nano science. AFM micro-cantilever (MC) accurate modeling would result in exact prediction of its vibrational behavior and also improve quality of images. Experimental usage of a model, requires a quantitative knowledge of its sensitivity to parameters variations. To identify the model’s behavior and its effective parameters in that, sensitivity analysis methods can be used. Regarding this, the non-uniform piezoelectric MC is modeled and simulated using finite element method based on the Timoshenko beam theory and the effect of environmental and geometric parameters on the MC’s vibrational amplitude have been investigated. The results show that the most influential parameter on amplitude, is the probe equilibrium distance, by increasing the humidity of the environment, the sensitivity of the amplitude increases to this parameter.

Keywords: AFM, Sensitivity Analysis, Timoshenko, finite element method.

Introduction:
Sensitivity analysis could be used for simplifying the model, increase the model’s reliability and show the connection between the input and output information. Up until now, lots of studies related to this issue have been done due to the importance of this. Lee and Chang studied the sensitivity analysis of a V-shaped AFM based on the couple stress theory [1]. Korayem and Damircheli investigated the sensitivity of higher modes of the MC compared to the surface hardness [2]. McCarthy and Mahmoudi analyzed the system parameters on non-linear response of the MC [3]. Mahmoudi and Jalili investigated the bending vibration of a piezo-electric MC experimentally and theoretically [4]. In this paper, Efast and Sobol methods were used to analyze the effects of environmental and geometric parameters on micro-cantilever’s amplitude.

Modeling the piezoelectric MC:
The MC was assumed as a four-layer beam with a rectangular cross-section based on Timoshenko beam theory, as shown in figure 1, and the mass of the probe’s tip was neglected. Also the probes width was assumed less in order to increase the cantilever’s sensitivity to the forces between probe’s tip and sample.

To extract the MC’s equations of motion, Hamilton’s Principle was used. Therefore, the kinetic and potential energy of each MC layers and external forces were extracted, and equations of motion have become coupled as equations
(1) and (2) according to the reference [5], so that the parameters correlated with the modified coupling stress theory are eliminated. In equations (1) and (2) terms $G$, $\rho$, $I$, $A$, $E$, $C$, $V$ and $K$ show shear modulus, density, inertia moment, cross section, elasticity modulus, coefficient of attenuation, voltage and shear coefficient (which is 0.83 for rectangular cross section), respectively. Constant coefficients for each section of the MC and also piezoelectric including $e_1$, $e_5$ and $\delta_{33}$ was calculated according to [5].

\[
\rho \left( \frac{\partial^2 \psi}{\partial t^2} \right) = (EI) \left( \frac{\partial^4 \psi}{\partial x^4} + \frac{\partial^4 \psi}{\partial y^4} \right) + KG \left( \frac{\partial \psi}{\partial x} \right) + I_p \left( \frac{\partial^2 \psi}{\partial x^2} \right)
\]

(1)

\[
\rho A \left( \frac{\partial^2 \psi}{\partial t^2} \right) - K A \left( \frac{\partial \psi}{\partial x} \right) - I_p \left( \frac{\partial \psi}{\partial x} \right) = 0
\]

(2)

Finite element method was used for discretization of equations of motion and the finite element coefficients have been calculated using the two sets of shape functions for the displacement caused by the slope and the deflection using the Galerkin equation [2]. Also, to solve the equations and calculating the natural frequencies, Laplace transforms were used. The Newmark method was used to calculate the amplitude, considering forces between the MC’s tip and the sample surface including Van der Waals, contact and capillary forces.

Analysis and results:

After calculation of the MC natural frequency, time response of the system was calculated through using the Newmark’s Algorithm. The amount of frequency and amplitude, where the MC is far from the surface, is obtained based on the Timoshenko beam theory which are 52.29 kHz and 3.79 µm, respectively. By comparing experimental results [4], 52.30 kHz and 3.75 µm, it is revealed that there is a high match between the theoretical and experimental results that could indicate high precision in modeling. When the MC is far from the surface, no force is applied to the probe. As the piezoelectric MC gets closer to the surface, the effects of the forces between the tip and the sample on the MC vibration behavior increases. According to Figure 2, as it gets closer to the sample surface, due to the enhancement of the interatomic forces, the MC vibration amplitude reduces. It is necessary to note that due to the changes of the natural frequency in various equilibrium distances, for each of them, the natural frequency associated with it is considered.

The force constants used here include water vapor-liquid energy ($\gamma_w$), radius of the probe (R), interatomic distance ($a_0$), water layer thickness (h), capillary bridge start ($d_{bn}$), breaking capillary bridge ($d_{off}$), the Hamaker coefficient (H), Needle Young’s Modulus ($E_t$), Sample Young’s Modulus ($E_s$), the MC Poisson coefficient ($\theta_t$), the Sample Poisson coefficient ($\theta_s$) and Quality factor (Q) which are equal to $75\text{mJ/m}^2$, 20nm, 0.34nm, 0.2nm, 0.4nm, 2.32nm, $6\times10^{-20}\text{J}$, 105Gpa, 78Gpa, 0.287, 0.287 and 167, respectively [5]. The sensitivity percentage of the piezoelectric MC amplitude to each of these constants is investigated through using Sobol and Efast.
methods of sensitivity analysis. Figure 3 depicts obtained results of amplitude sensitivity analysis. Both methods by small disparity, lower than 10%, indicate that the most sensitive parameter is the MC equilibrium distance from the sample surface. As it showed, rising in the equilibrium distance leads to decline in the effect of the tip-sample forces which causes the amplitude increase. In addition, results show that sample surface module of elasticity has the least impact on the amplitude.

**Conclusion**

The results show, the less distance between the probe and sample surface, the more effect of tip-sample forces which leads to decrease the piezoelectric MC vibration amplitude. Also, among the effective parameter, the MC equilibrium distance has the most impact on the vibration amplitude. Additionally, as the humidity of the environment declines, the effect of Hamaker constant and probe radius has climbed up while, the impact of other factors such as equilibrium distance have declined.

**References**


